Jacobs University Bremen

Final Paper

Modeling Distribution Networks using the Nagel-Schreckenberg Traffic Model and Varying Junction Rules

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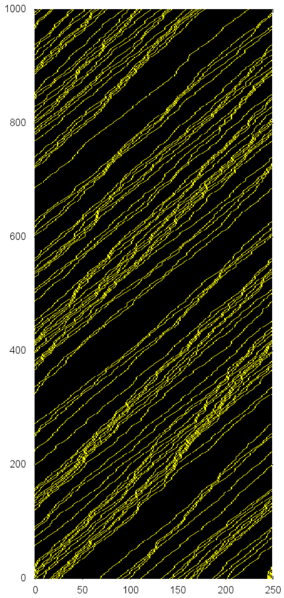
Contents

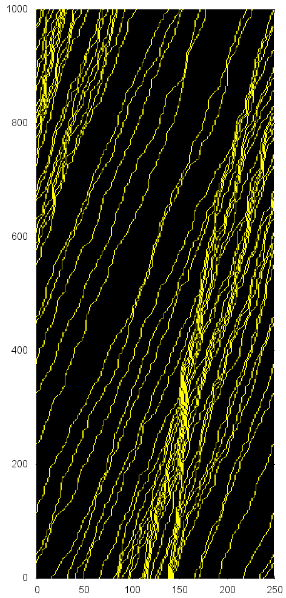
List of Figures

1 Introduction

At the heart of the wide and diverse field of Logistics lies the transport of goods. This transport occurs at a multitude of scales and between multiple locations of varying distances. But the experimental behavior of such dynamics is difficult to capture. The theory of traffic simulation has been thoroughly investigated, for example by Wolfram and his Rule 184 of cellular automata, as well as by the famous Nagel-Schreckenberg model [9, 10]. These models of Traffic Flow have been shown to be extremely accurate in predicting the occurrence and behavior of traffic jams dependent on initial conditions, density, and driver behavior. By driver behavior we are specifically refering to the Braking probability that is an essential part of the Nagel-Schreckenberg model; the Braking Probability creates the instabilities that give rise to random traffic jams within a one dimensional closed street. Specifically it defines how often any given car will slow down without reason.



(A) Simulation of Street with Car Density of 20% and braking probability of 30%



(B) Simulation of Street with Car Density of 20% and braking probability of 70%

FIGURE 1: Two simulations were produced using the Nagel-Schreckenberg model with varying Braking Probabilities. The horizontal axes represent the space domain while the vertical axes show the change in the systems with respect to time. One should note that the first iteration is at the bottom. [9]

In Figure 1 we notice the diagonal lines characteristic of Traffic Flow simulations using the Space-Time plot style. The aggregation of cars, here depicted as yellow pixels, is due to the Braking Probability described above. The reason the plot on the left has a larger build-up of cars, or a traffic jam, is specifically because its Braking Probability is much greater.

While these models help to investigate 1-Dimensional traffic, for complex distribution networks we need a 2-Dimensional model with the same efficacy. But the inclusion of nodes creates a modeling discrepancy as various rules for the behavior of the model at these nodes/junctions exist.[3, 7] Work done by Brockfeld et al. [1] and Chopard et al. [2] describe the dynamics on a square grid of streets with varying densities and intersection rules. Namely, they implement right priority intersections and typical stop light intersections. These models are quite useful in areas of normalized street length and intersection, but in the case of Distribution Networks they are not as applicable due to the varying degrees of connectivity at each node as well as the possible variation in distances between nodes (distribution centers). These network models would simulate the transport of goods between distribution centers, thus giving insight into the organization of distribution networks.

2 Simplifications

As distribution networks vary greatly in size, connectivity, and number of nodes we approach the problem from a simplified viewpoint. First, we assume all distances between distribution centers to be equal. While this assumption is made throughout the paper it is worthwhile to note that individual lengths could be easily changed in the proposed model, it would just require additional adjustments to the collision and streaming steps. This assumption was made primarily to maintain consistency in the model and to allow for simpler analysis of the results. Secondly, each connection between nodes is assumed to run both ways. This is equivalent to having two-way streets.

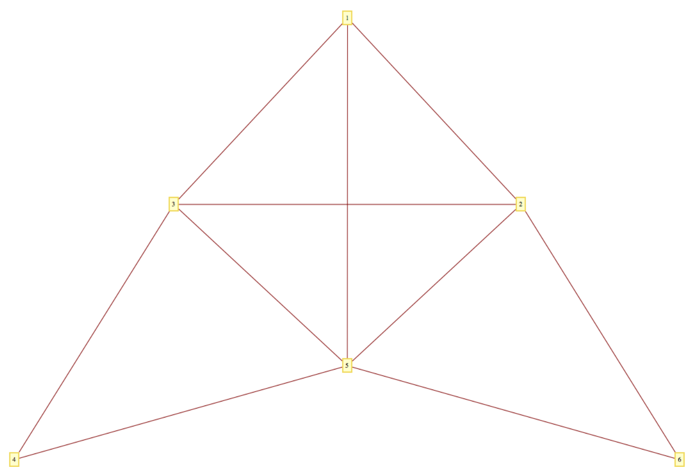


FIGURE 2: This is the simplified Distribution Network that was analyzed. It contains 6 nodes with varying degrees of connectivity.

Figure 1 could be considered a simulation on the network containing only two nodes, and where *Traffic Jams* are those areas of high pixel density on any given line and consequently the evolution of this line over time.

3 Implementation

The results seen in Figure 1 were produced from code written in C++, from that point the code was expanded to allow for multiple streets. The Collision and Streaming steps described in the Nagel-Schreckenberg model [9] are carried out on each "street" independently and simultaneously relative to the rest of the code. Now the more difficult part arises, how to couple streets together using junction rules.

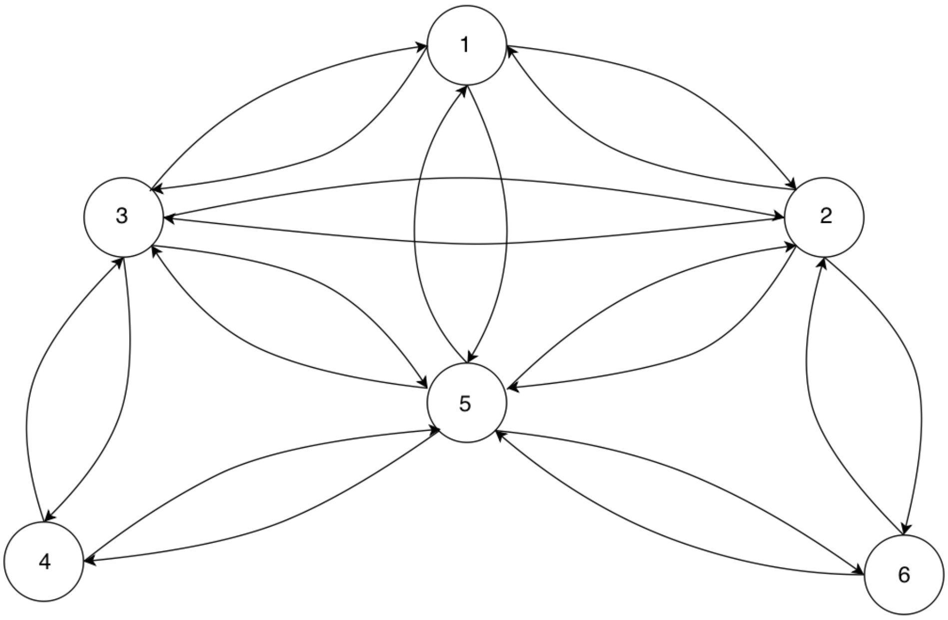


FIGURE 3: Network with directed Streets

Expanding from the network seen in Figure 2 we create the network in Figure 3 with all possible streets of simulation. There are 20 streets in total. Because this is a directed graph one will notice that cycles are easily found.

3.1 Junction Rules

Within the network seen in Figure 3 one can find cycles of varying lengths. These cycles could be interpreted as the areas where cars will travel, there could be an even distribution, whereas cars are distributed to random streets the moment they reach a node, or a fixed rotary junction where each given street is forced to go to pair according to the cycle. Firstly, we are interested in a cycle which contains all streets. Because of the parity of the network in Figure 3 we are able to find such a cycle. Using the labels in Figure 3 we can explicitly define the order of nodes that a car would pass through. That order is:

$$1 \to 2 \to 6 \to 5 \to 4 \to 3 \to 1 \to 5 \to 6 \to 2 \to \tag{1}$$
$$5 \to 3 \to 4 \to 5 \to 1 \to 3 \to 2 \to 3 \to 5 \to 2 \to 1$$

Secondly, a randomized junction rule was created to see the dynamics in comparison to the initial rotary junction. This junction is very simply implemented, one gives equal probability to each outgoing street at an intersection and once a car reaches a node it is distributed to a random street.

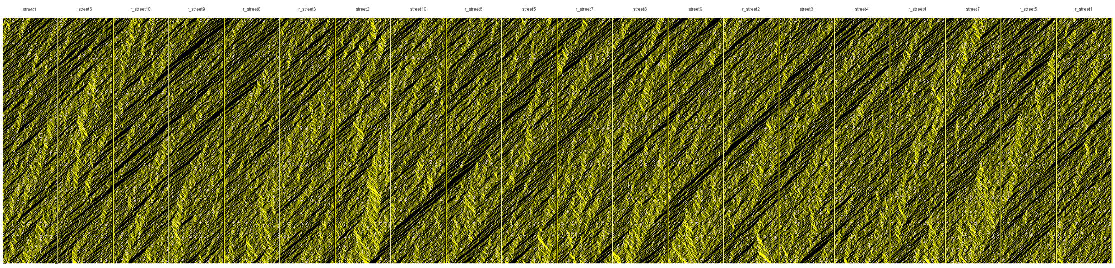
Lastly, I implemented a junction rule that combines the original two approaches. This mixed junction rule implements a randomized junction at the node with the highest degree of connectivity, and creates small cycles of three nodes with the rest of the streets. Namely, it creates cycles between nodes 1, 2, and 3 as well as nodes 3, 4, and 5 and lastly nodes 2, 5, 6 are also connected.

4 Results

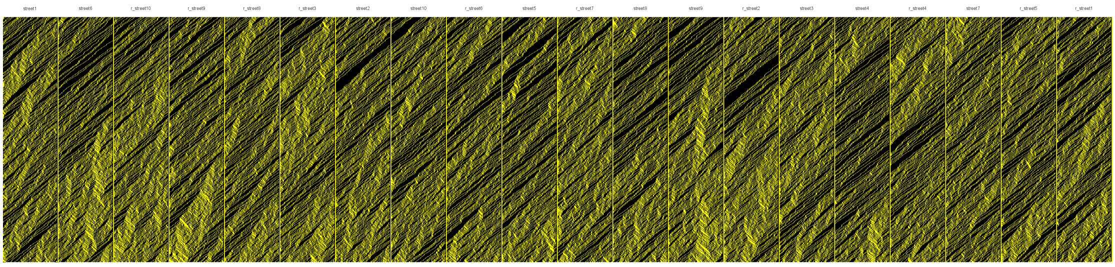
To be able to consistently visualize the results and accurately differentiate between various simulations we plot the results of the distribution network similarly to the plot in Figure 1. Namely, a Space-Time Plot is created. The issue is in what order to connect the *streets*, and one logical order is the one seen in Equation 1. This means that we expect the plot of the Rotary Junction Rules to look like one large street with breaks shown by vertical lines.

As expected, using this type of visualization clearly shows the propagation of traffic jams across nodes, as seen in Subfigure (A) of Figure 4. Additional results with greater Braking Probabilities were also carried out.

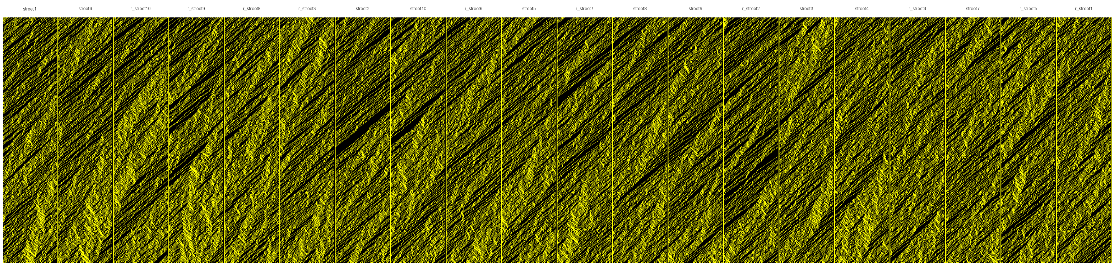
Simulations using a lower Density were also produced, to verify that the dynamics seen were not only due to the high density of cars present.



(A) Rotary Junction

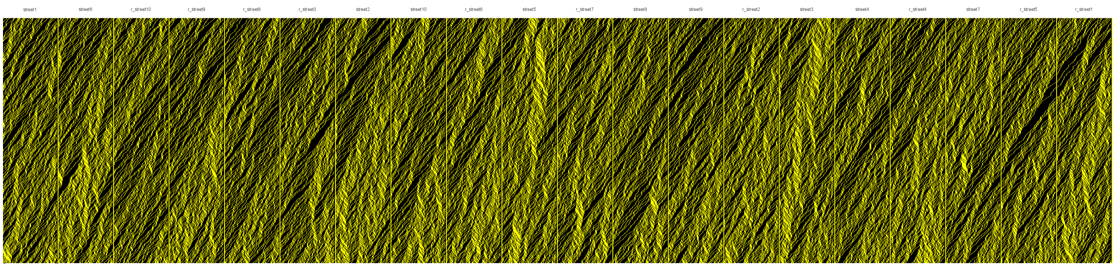


(B) Random Junction

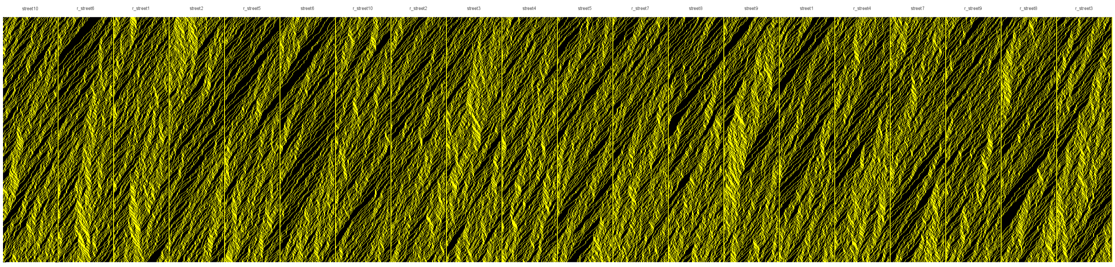


(C) Mixed Junction

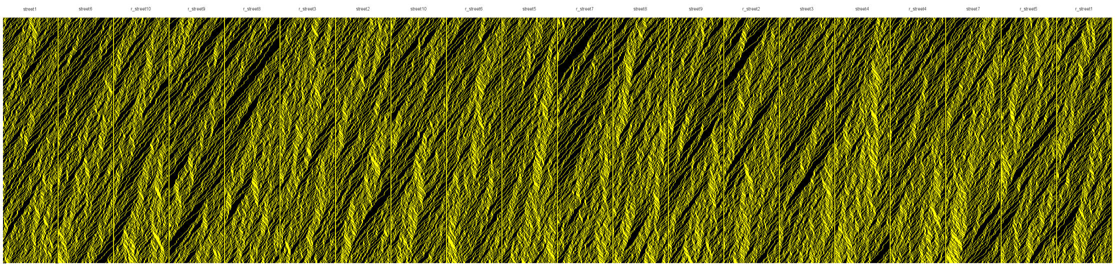
Figure 4: Simulations carried out with a Braking Probability of 25% and Density of 50%



(A) Rotary Junction

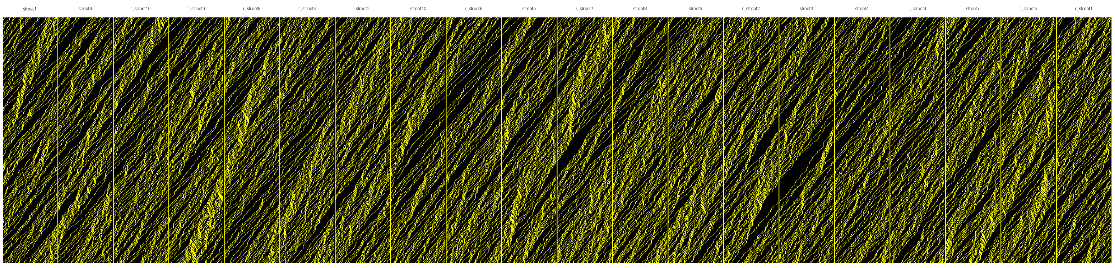


(B) Random Junction

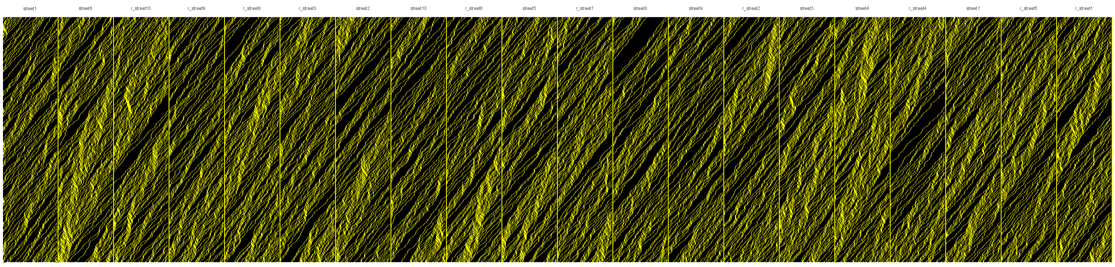


(C) Mixed Junction

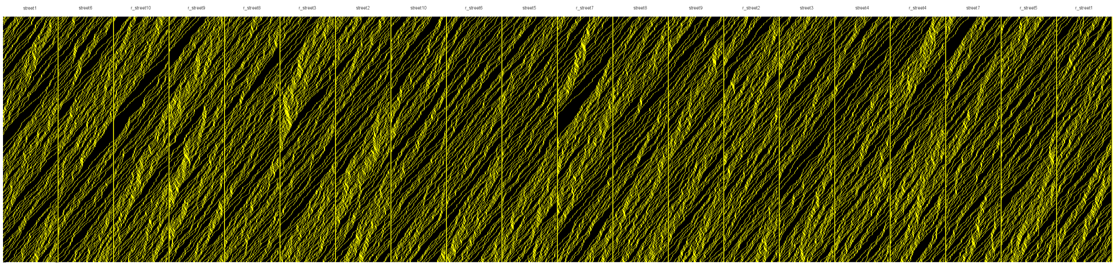
FIGURE 5: Simulations carried out with a Braking Probability of 50% and Density of 50%



(A) Rotary Junction



(B) Random Junction



(C) Mixed Junction

FIGURE 6: Simulations carried out with a Braking Probability of 50% and Density of 30%

5 Conclusions

From the results found we noticed the largest amount of traffic jams present in the Rotary Junction, and that the Random and Mixed Junctions seemed to produce similar results. From this we can infer that within a distribution network it is always good to maintain a reasonable degree of randomness. Forcing shipments to travel along predetermined paths causes blockages that cannot be easily fixed. If the system has certain degree of inherent randomness it greatly improves the ability of the network to evenly distribute the high density of cars needed.

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